General Certificate of Education
January 2005
Advanced Subsidiary Examination

MATHEMATICS **MPC2**
Unit Pure Core 2

Friday 21 January 2005 Afternoon Session

In addition to this paper you will require:
- an 8-page answer book;
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC2.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

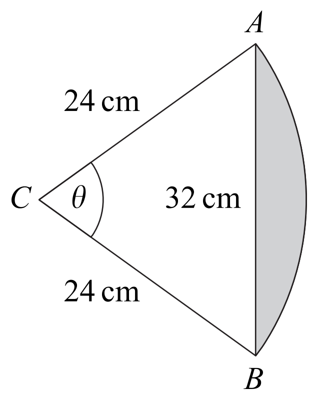
1 A curve is defined for $x > 0$ by the equation $y = x + \dfrac{2}{x}$.

(a) (i) Find $\dfrac{dy}{dx}$. *(3 marks)*

(ii) Hence show that the gradient of the curve at the point P where $x = 2$ is $\frac{1}{2}$. *(1 mark)*

(b) Find an equation of the normal to the curve at this point P. *(4 marks)*

2 The diagram shows a triangle ABC and the arc AB of a circle whose centre is C and whose radius is 24 cm.



The length of the side AB of the triangle is 32 cm. The size of the angle ACB is θ radians.

(a) Show that $\theta = 1.46$ correct to three significant figures. *(3 marks)*

(b) Calculate the length of the arc AB to the nearest cm. *(2 marks)*

(c) (i) Calculate the area of the sector ABC to the nearest cm^2. *(2 marks)*

(ii) Hence calculate the area of the shaded segment to the nearest cm^2. *(3 marks)*

3 An arithmetic series has fifth term 46 and twentieth term 181.

(a) (i) Show that the common difference is 9. *(3 marks)*

(ii) Find the first term. *(1 mark)*

(b) Find the sum of the first 20 terms of the series. *(2 marks)*

(c) The nth term of the series is u_n.

Given that the sum of the first 50 terms of the series is 11525, find the value of

$$\sum_{n=21}^{50} u_n$$ *(2 marks)*

4 (a) Write $\sqrt{x}$ in the form x^k, where k is a fraction. *(1 mark)*

(b) Hence express $\sqrt{x}(x-1)$ in the form $x^p - x^q$. *(2 marks)*

(c) Find $\int \sqrt{x}(x-1)\,\mathrm{d}x$. *(3 marks)*

(d) Hence show that $\int_1^2 \sqrt{x}(x-1)\,\mathrm{d}x = \frac{4}{15}\left(\sqrt{2}+1\right)$. *(3 marks)*

5 (a) Given that

$$\log_a x = 3\log_a 6 - \log_a 8$$

where a is a positive constant, show that $x = 27$. *(3 marks)*

(b) Write down the value of:

(i) $\log_4 1$; *(1 mark)*

(ii) $\log_4 4$; *(1 mark)*

(iii) $\log_4 2$; *(1 mark)*

(iv) $\log_4 8$. *(1 mark)*

Turn over ▶

6 (a) (i) Using the binomial expansion, or otherwise, express $(2+x)^3$ in the form $8 + ax + bx^2 + x^3$, where a and b are integers. *(3 marks)*

(ii) Write down the expansion of $(2-x)^3$. *(2 marks)*

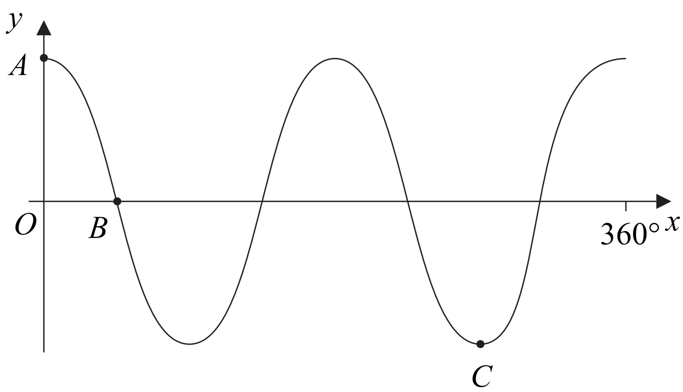
(b) Hence show that $(2+x)^3 - (2-x)^3 = 24x + 2x^3$. *(2 marks)*

(c) Hence show that the curve with equation

$$y = (2+x)^3 - (2-x)^3$$

has no stationary points. *(3 marks)*

7 The diagram shows the graph of $y = \cos 2x$ for $0° \leqslant x \leqslant 360°$.



(a) Write down the coordinates of the points A, B and C marked on the diagram. *(4 marks)*

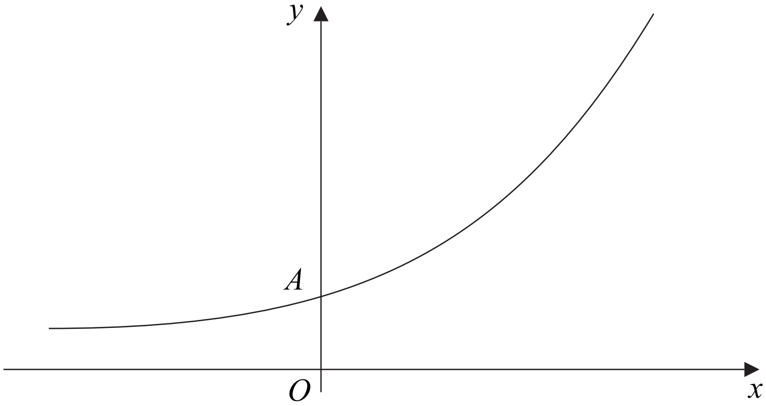
(b) Describe the single geometrical transformation by which the curve with equation $y = \cos 2x$ can be obtained from the curve with equation $y = \cos x$. *(2 marks)*

(c) Solve the equation

$$\cos 2x = 0.37$$

giving all solutions to the nearest 0.1° in the interval $0° \leqslant x \leqslant 360°$. (No credit will be given for simply reading values from a graph.) *(5 marks)*

8 The diagram shows a sketch of the curve with equation $y = 3^x + 1$.



The curve intersects the y-axis at the point A.

(a) Write down the y-coordinate of point A. *(1 mark)*

(b) (i) Use the trapezium rule with five ordinates (four strips) to find an approximation for $\int_0^1 (3^x + 1)\,dx$, giving your answer to three significant figures. *(4 marks)*

(ii) By considering the graph of $y = 3^x + 1$, explain with the aid of a diagram whether your approximation will be an overestimate or an underestimate of the true value of $\int_0^1 (3^x + 1)\,dx$. *(2 marks)*

(c) The line $y = 5$ intersects the curve $y = 3^x + 1$ at the point P. By solving a suitable equation, find the x-coordinate of the point P. Give your answer to four decimal places. *(4 marks)*

(d) The curve $y = 3^x + 1$ is reflected in the y-axis to give the curve with equation $y = \mathrm{f}(x)$. Write down an expression for $\mathrm{f}(x)$. *(1 mark)*

END OF QUESTIONS

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